Extended Abstract: Simulating Quantum Magnetism on Noisy Quantum Computers: An Analysis of Trotter-Suzuki and qDRIFT

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***Abstract*—We examine the performance of two quantum algorithms for simulating quantum magnetism: First Order Trotter-Suzuki decomposition (FOTS) [1] and quantum stochastic drift protocol (qDRIFT) [2]. FOTS is a deterministic algorithm for simulating quantum dynamics, and qDRIFT is a recently developed algorithm utilizing randomization to speed up the computation of Hamiltonian simulations. We investigate the accuracy of qDRIFT when compared to FOTS with and without noise for simulating the Ising and Heisenberg models of quantum magnetism. We simulated the dynamics of a uniformly interacting one-dimensional Ising model with a set Hamiltonian for combinations of fixed or randomized initial state and observable. The algorithms were created and tested using the Qiskit library in Python [3]. When the initial state and observable were fixed, qDRIFT on noiseless simulation had greater algorithmic error than FOTS. On the IBM Quantum devices [4], the difference in accuracy between qDRIFT and FOTS was lower than on simulation. This indicates that while the efficacy of qDRIFT is worse without noise for the models we studied, the results from qDRIFT are less affected in the presence of noise. When randomizing the initial state and observable on hardware, qDRIFT and FOTS had nearly identical accuracy. However, when the Heisenberg model had one dominant ferromagnetic coupling interaction and other weaker interactions, qDRIFT outperformed FOTS.**

I. INTRODUCTION

This project began as part of the Quantum Education in Science and Technology program within the Johns Hopkins University Applied Physics Laboratory ASPIRE program. We were introduced to theoretical topics in quantum information and languages for programming cloud-based quantum computers. Quantum computing is a new computing paradigm that uses quantum mechanics to perform computations. This technology has the potential to efficiently solve a variety of challenging problems, such as optimization, breaking encryption, and simulating molecules relevant to drug development and materials discovery. However, quantum computers are currently limited by noise generated by unwanted environmental interactions, making qubit operations error-prone. Therefore, it is critical to develop algorithms that can effectively take advantage of the ability of quantum computers to accelerate simulations of quantum mechanical systems even when affected by noise. Our focus was on algorithms for material simulation, particularly quantum magnetism. We aimed to evaluate one such novel algorithm, quantum stochastic drift protocol (qDRIFT), and compare it with an established algorithm, First Order Trotter-Suzuki decomposition (FOTS), and identify optimal experimental usage for simulating quantum systems on current quantum hardware.

II. BACKGROUND

The dynamics of a quantum system are governed by a Hamiltonian that generates the time-evolution operator. Quantum algorithms for simulating such systems aim to directly generate the time-evolution operator as a quantum circuit that can be programmed on a quantum computer. FOTS utilizes the sum representation of a Hamiltonian to deterministically decompose the time-evolution operator into a series of one and two qubit-gates. However, the gate count for FOTS, and therefore error, scales with the number of terms in the Hamiltonian, and chemical systems can have millions of terms [1]. qDRIFT is a novel compilation routine that simulates quantum systems by weighting operations based on their importance to the simulation accuracy. It could potentially offer drastic improvements to FOTS as its gate count scales with the sum of coefficients of Hamiltonian [1], [2].

III. PROCESS

We analyzed the Ising chain with a set Hamiltonian for several scenarios: fixed time of evolution with a

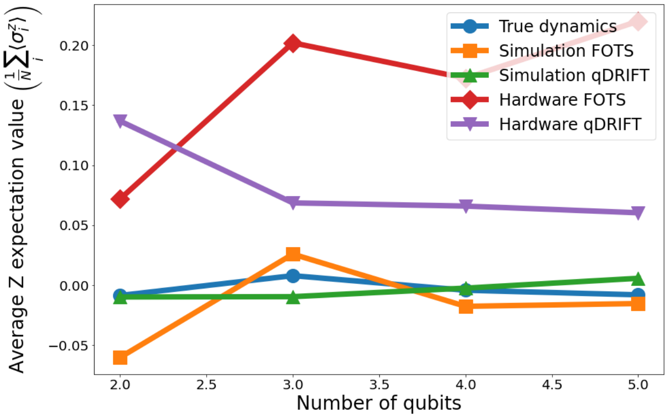

Fig. 1. Heisenberg spin chain comparison at time = 0.5. Parameters: $J_x, J_y, J_z = \frac{0.1}{N-1}, \frac{10}{N-1}, \frac{0.05}{N-1}$, where J is a coupling constant and N is the particle count.

fixed initial state and observable, randomized initial state and fixed observable, and randomized initial state and observable. To do this, we created and tested circuits for the qDRIFT and FOTS protocols using Qiskit [3]. Performing these various tests provides a more holistic understanding of different circumstances that could impact the performance of qDRIFT and FOTS. Unlike FOTS, qDRIFT is stochastic, so we averaged the output over many instances of random circuits to improve accuracy. The number of gates was kept constant between the two methods to make a fair comparison. We tested the algorithms using classical numerical simulations, quantum circuit simulators, and quantum computer hardware such as *ibmq_manila* and *ibm_nairobi* [4]. We used statistical analysis to determine error rates and discrepancies between true, simulation, and hardware values.

IV. RESULTS

Table I shows how we found qDRIFT's error to be 3-7 times greater than that of FOTS during simulation. Errors for both algorithms decreased drastically with more randomization for the initial state and observable. For our hardware tests with a set initial state and observable, neither algorithm was very accurate, and qDRIFT had 32.5% more error than FOTS. Since simulation and hardware outputs were quite similar for qDRIFT, we compared the error of the qDRIFT algorithm with the difference in simulation and hardware outputs for qDRIFT. We found that the difference between simulation and hardware output was only low when the algorithmic error was high. When randomized states were introduced to hardware, the mean error dropped for both algorithms, as in the simulations. Notably, qDRIFT's error on hardware dropped by 69.0% compared to when both the initial state and observable were set. Still, on average, qDRIFT's error was 24.8% higher than FOTS's error. After the initial state and observable were randomized, both algorithms' errors on hardware were further reduced, and the error of qDRIFT was only 8.0% higher than the error of FOTS. Furthermore, we found some parameter combinations for the Heisenberg model where qDRIFT outperforms FOTS on both hardware and simulation. These Hamiltonians had a single interaction term that was much larger than the other terms. Fig. 1 shows how the difference in accuracy between qDRIFT and FOTS increased with qubit count when simulations of these Hamiltonians were conducted on hardware.

TABLE I
MEAN ERROR OF FOTS AND QDRIFT WITH DIFFERENT TESTING CONDITIONS

Testing Conditions	**Mean Error (Simulation)**	**Mean Error (Hardware)**
Set state and observable	FOTS: 0.068 qDRIFT: 0.479	FOTS: 0.431 qDRIFT: 0.571
Random state and set observable	FOTS: 0.052 qDRIFT: 0.171	FOTS: 0.142 qDRIFT: 0.177
Random state and observable	FOTS: 0.042 qDRIFT: 0.158	FOTS: 0.130 qDRIFT: 0.140

V. FUTURE WORK

The robustness of publicly available quantum hardware limited the size of the Ising models we investigated to a maximum of five qubits. Future work will examine qDRIFT when applied to more complex systems like molecules relevant to quantum chemistry, which have many more Hamiltonian terms and few dominant interaction terms. This would require higher-qubit quantum devices. Nonetheless, we demonstrated the impact of noise even on low-qubit quantum computers and that implementing Hamiltonian simulation necessitates effective noise management.

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